**NTU SSS Economics HE2001**  
**Tutorial 9 (Perfect Competition and Welfare)**

1) Consider an Edgeworth box with 2 goods, apples and oranges.   
There are two consumers Alice and Bob. Alice has an endowment of 10 oranges while Bob has an endowment of 10 apples. Both Alice and Bob have Cobb-Douglas Utility .

a) For prices and , derive the demands of Alice and Bob.

Solving the utility maximisation problem (MRS=price ratio and the budget constraint), each of them spends half their budget on oranges and apples.

Hence, we have:

Alice’s demand for apples, is .

Bob’s demand for apples is .

Likewise,

Alice’s demand for oranges, is .

Bob’s demand for oranges is .

b) Given the above demands which are a function of prices, write out the condition for market clearing.

The market clearing condition for apples is

Or .

This can be simplified to give .

Doing this for oranges will also give you the same answer.

c) Show that the competitive equilibrium allocation in this exchange economy is where Alice and Bob have 5 oranges and 5 apples each. Illustrate the competitive equilibrium in an Edgeworth Box, showing how it is pareto efficient.

If , substituting into their demands, each of them demands 5 oranges and 5 apples each and both markets clear. Thus, we have shown that Alice and Bob having 5 oranges and 5 apples each and form a competitive equilibrium I.e., maximising their utility relative to these prices, their demands are indeed (5,5) and markets clear.

The figure will look similar to that shown in your lecture, just that the tangency point is (5,5).

d) Show that the allocation of 9 oranges and 9 apples to Alice and 1 orange and 1 apple to Bob is also pareto efficient. (Hint: try drawing the indifference curves through the point)

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(The flipped answer is also correct)

When the MRS is the same, it will be a pareto efficient outcome: you can check that the allocation gives a value of -1 for both.

e) Give two examples of a redistribution of the original endowment which will result in the same pareto efficient outcome in a competitive equilibrium.

Any endowment which lies on the above price line passing through will result in the same competitive equilibrium. For example, or .

[With Bob having an endowment of the rest of the apples and oranges.]

2) Linus Straight’s utility function is , where a is his consumption of apples and b is his consumption of bananas. Lucy Kink’s utility function is . Lucy initially has 12 apples and no bananas. Linus initially has 12 bananas and no apples.

1. Draw an Edgeworth box, showing the initial allocation and sketching in a few indifference curves. Measure Lucy’s consumption from the upper right corner and Linus’s from the lower left corner. Draw a line through all of the Pareto optimal allocations.

图表

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If is not equal to 2 for Lucy, then giving the extra to Linus will mean that both can be better off. Graphically, this means that indifference curves passing through any point not on the green line will have an area of pareto improvement between them.

1. In this economy, in competitive equilibrium, what will be the ratio of the price of apples to the price of bananas ?

This has to hold, otherwise, there will always be excess demand. At this price ratio, Linus is indifferent between any bundle in his budget.

If , then . This implies that and that apples have a lower marginal utility per unit cost always. Hence, Linus will demand only bananas=12. Excess demand as Lucy also has positive demand of bananas.

If then . This implies that and that apples have a higher marginal utility per unit cost always. Hence, Linus will demand only apples. . Excess demand on apples.

Try drawing out these different scenarios in a diagram and see where is the highest indifference curve at which Linus can consume.

1. Find the quantities of apples and bananas consumed in competitive equilibrium by Linus and Lucy.

Let be Lucy’s consumption of apples and let be her consumption of bananas. To maximize her utility, she must consume . This gives us one equation in two unknowns.

To find a second equation, consider Lucy’s budget constraint, which gives . Substitute into the second equation. Finally, we can solve .

For Linus, he consumes units of apples and unites of bananas.

This costs which is indeed equal to the value of Linus’s endowment.

For Linus, he consumes units of apples and unites of bananas.

3) Paul and David consume apples and oranges. Paul’s utility function is and David’s utility function is , where and are apple consumptions for Paul and David, and and are orange consumptions for Paul and David.

There are a total of 12 apples and 12 oranges to divide between Paul and David.

(a) Draw an Edgeworth box showing some of their indifference curves. Mark the Pareto optimal allocations on your graph.

C

Oranges

David

Pareto optimal allocations in orange.

Apples

David’s indifference curves

D

A

Paul’s indifference curves

B

Paul

Any point in the interior of the edgeworth box or to the left and top is pareto improved on by some other allocation (to their bottom right). For example, point A in the interior is pareto improved on by point B. Points C and D on the top and left are pareto improved on by B as well.

Any point on the bottom or right border cannot be pareto improved on by anything in the edgeworth box (i.e. a feasible allocation).

(b I & II) Write one inequality that says that Paul likes his own bundle as well as much as he likes David’s and write another inequality that says that David likes his own bundle as much as he likes Paul’s.

(b III) Use the fact that at feasible allocations, and to rewrite the above inequalities in terms of and for Paul and and for David.

Just substitute these equalities in the question into the respective inequalities in b) to obtain:

(b IV and C)

(1) The allocations where Paul prefers his own allocation to David’s: Blue area

(2) The allocations where David prefers his own allocation to Paul’s: Red Area

(3) Envy free allocations: Purple Area

(4) Fair allocations: Orange lines.

Chart, pie chart

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4) Consider a small exchange economy with two consumers, Astrid and Birger, and two commodities, herring and cheese. Astrid’s initial endowment is 4 units of herring and 1 unit of cheese. Birger’s initial endowment has no herring and 7 units of cheese. Astrid’s utility function is . Birger is a more inflexible person. His utility function is .

(Here and are the amounts of herring and cheese for Astrid, and and are amounts of herring and cheese for Birger.)

(a) Draw an Edgeworth box, showing the initial allocation and sketching in a few indifference curves. Measure Astrid’s consumption from the lower left and Birger’s from the upper right. In your Edgeworth box, draw two different indifference curves for each person.

(b) Illustrate on the same diagram, the locus of Pareto optimal allocations.

Notice that if in an allocation, , then it is not a pareto optimal allocation, as some of the excess can be given to Astrid without hurting Birger. That new allocation would pareto improve on the initial allocation.

All allocations where are pareto optimal as the indifference curves are tangent to each other at all these points. Hence there are no allocations which pareto improve on them.

Chart, line chart

Description automatically generated

Solid Black line: Pareto Optimal Allocations

The pareto set also includes the black horizontal line from 0 to 4 cheese. There is are no pareto improvements for these allocations.

This is because the indifference curve for 0 utility for Astrid is different in shape from the blue indifference curves. More accurately it is the green L shaped line as shown below. Why? This is because if Astrid has either 0 cheese or 0 herring, utility will always be equals to 0. This means that we cannot actually find a pareto improvement on the horizontal black line.

This is just a peculiarity of the utility function: if Astrid is consuming 0 herring, then even if we give him more cheese (for which Birger may have excess), he still doesn’t get more utility.

(c) Let cheese be the numeraire (with price 1) and let denote the price of herring. Derive the allocations and prices in the competitive equilibrium. (Hint follow the steps in the lecture and question 1)

(I) Birger will demand an equal amount of herring and cheese as this is the most efficient use of his income. This will cost . Since his income is given that he owns units of cheese, he will hence demand units of herring (and cheese)

(II)Since this is a cobb douglas utility, Astrid will spend half of his budget on herring and half on cheese (since the exponents are the same). He will hence demand units of herring.

Alternatively, you can equate MRS=price ratio and use the budget constraint to get the final answer.

See <https://en.wikipedia.org/wiki/Cobb–Douglas_production_function> for more on cobb douglas utility.

(III)

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There is equilibrium when the demand for herring equals to the supply of herring in the economy.

This is when

Substitute this into the demand functions to get their quantities demanded at this price.

Notice that the competitive equilibrium is pareto efficient.

*(Note that you can follow the same procedure but for cheese instead and you will get the same answer. The reason that we only need to solve 1 market is because when there is market clearing in one market, there will automatically be market clearing in the other market if consumers exhaust their budgets.)*

**Sample Question (No solutions will be provided for these)**

Consider a small exchange economy, with two consumers, Iris and Joseph, and two goods and . Iris is endowed with 15 units of and 3 units of . Joseph is endowed with 5 units of and 7 units of .

Iris has utility while Joseph has utility .

1. Draw the Edgeworth box for this exchange economy as accurately as possible, indicating the endowment and drawing several indifference curves for Iris and Joseph. **(7 marks)**
2. In the figure you have drawn in (a), draw the indifference curves through the endowment and indicate where a competitive equilibrium must lie. Explain. **(7 marks)**
3. Is the allocation where Iris gets while Joseph gets a fair allocation? Explain.  **(8 marks)**
4. “There always exists an initial endowment for Iris and Joseph where the allocation in the consequent competitive equilibrium is fair for any (standard) preferences of Ivy and Joseph.” True or False? Explain. **(8 marks)**